

# MODE FORMATION AND BIFURCATION IN TAYLOR VORTEX FLOW WITH VERY SMALL ASPECT RATIO

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We investigate the unsteady Taylor vortex flow in a concentric cylinder system where the cylinder length is very short. The inner cylinder is rotating, and the outer cylinder and end walls of cylinders are stationary. This flow appears in journal bearings, a variety of fluid machinery and chemical reactors. When the flow pattern changes, the unsteady variations of property values such as torque and rate of reaction arise. Therefore the analysis and prediction of the unsteady flow is important from the engineering point of view.

It is well known that the Taylor vortex flow has three modes: primary mode, normal secondary mode and anomalous secondary mode. When the end of walls of cylinders is fixed, the normal mode has normal cells give an inward flow near the end wall. The anomalous mode has anomalous cell(s) on both or either end walls. The anomalous cell gives an outward flow near the end wall, which is opposite to the flow direction in the normal modes.

The main parameters which determine modes of Taylor vortex flows are the Reynolds number  $Re$  and the aspect ratio  $\Gamma$ . With the characteristic length being the gap length between cylinders, the Reynolds number is defined by the rotation speed and the aspect ratio is a dimensionless length of cylinders. When the aspect ratio is of order of unity, experimental and numerical evidence has shown the Taylor vortex flow has the one-cell mode and the two-cell mode. The existence of a twin cell which has a detaching point and an attaching point on both end walls has been found experimentally, but it has not been predicted numerically. In present numerical study, the existence of the one-cell, the two-cell and the twin-cell mode is confirmed, and the investigation is carried out on the decelerating Taylor vortex flows.

The governing equations are unsteady axisymmetric Navier-Stokes equations and they are solved by the finite difference methods. The end of walls of cylinders is fixed.

The boundary condition on velocity components is a non-slip condition and the initial velocity is zero in a whole domain.

Figure 1 shows a table of a mode pattern. The aspect ratio is from 0.1 to 1.6 with an interval of 0.1 and the Reynolds number is from 100 to 1500 with an interval of 100. In the region denoted by "Rotation", the flows are time dependent.

The computational results of the velocity vectors in the normal two-cell mode and the asymmetric normal two-cell mode are shown in Figure 2, where the inner cylinder is on the left. In Figure 2(a), the normal two-cell mode gives inward flow near the end walls and outward flow near the mid-plane in the axial direction. Figure 2(b) shows the asymmetric normal two-cell mode which is indicated by asterisk in Figure 1.

The comparisons of experimental and numerical results of the anomalous one-cell mode and the twin-cell mode are shown in Figure 3, Figure 4, respectively. In Figure 3, the anomalous cell which rotates in the counterclockwise direction accompanies with extra cells which rotate in the clockwise direction at the inner cylinder side and the outer cylinder side, though the outer extra cell is not clearly shown in the figure of velocity vectors. In Figure 4, the twin-cell mode has the separation points on the upper cylinder edge and the lower cylinder edge, while the normal two-cell mode has the separation points on the inner cylinder side and the outer cylinder side. The numerical prediction of existence of the twin-cell mode have been done. This fact demonstrates the validity of this calculation.

When the rotation speed of the inner cylinder decreases, transitions from a certain mode to another mode are found. Figure 5 shows the variation of the stream-function contour during the mode exchange process from the anomalous one-cell mode to the normal two-cell mode. As the rotation speed decreases, the extra cells merge with each other and

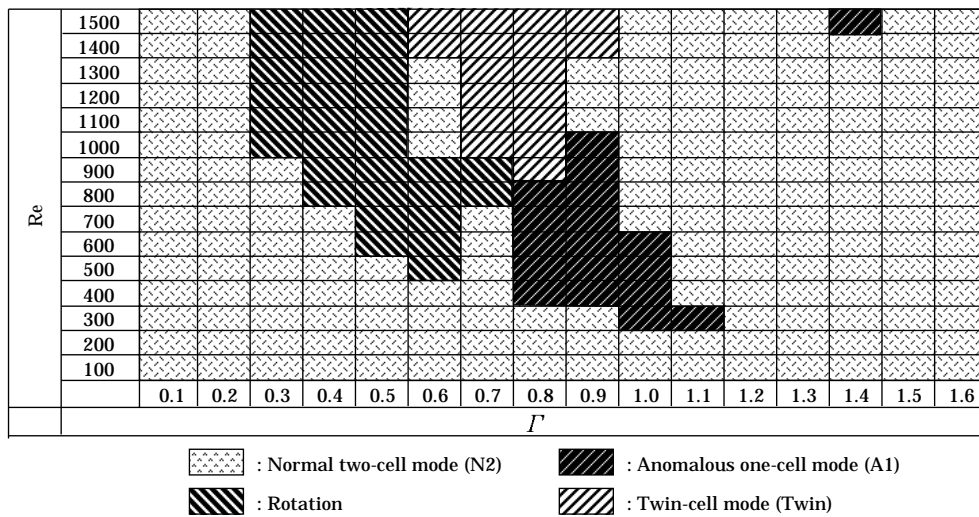


Figure 1. Numerically calculated mode pattern.  $Re$  is increased suddenly from rest.

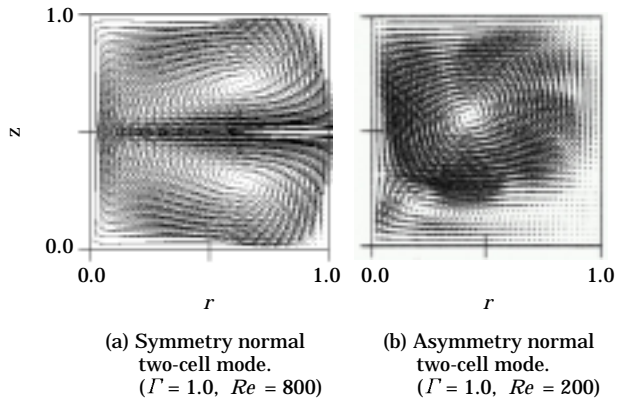


Figure 2. Velocity vectors in normal two-cell mode

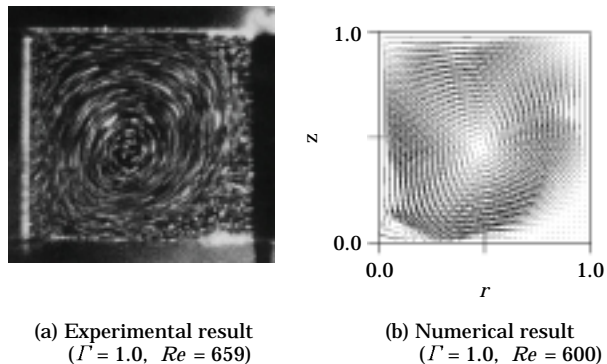


Figure 3. Anomalous one-cell mode

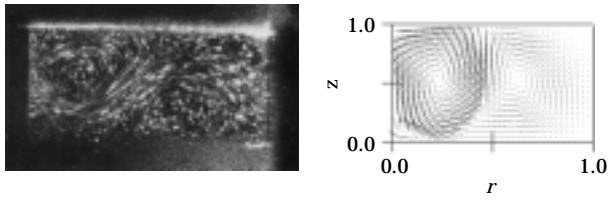
form one new normal cell, and the flow becomes the normal two-cell mode.

Figure 6 shows the mode exchange process from the normal two-cell mode to the anomalous one-cell mode. As the rotation speed decreases, one cell grows up. The other cell is pushed aside and it is divided into two extra cells: one is on the inner cylinder and the other is on the outer cylinder. The flow finally becomes the anomalous one-cell mode.

Figure 7 shows the mode exchange process from the twin-cell mode to the anomalous one-cell mode. An extra cell is formed around the inner corner where  $r = 0.0, z = 0.0$ . As the rotation speed decreases, the separation point

on the end wall opposite to the end wall to which the extra cell attaches gradually shifts outward, and it moves onto the outer cylinder. Then the flow becomes the anomalous one-cell mode.

The unsteady Taylor vortex flows with very small aspect ratio in a concentric cylinder system have been investigated, and the table of a mode pattern has obtained when the aspect ratio is from 0.1 to 1.6 with an interval of 0.1 and the Reynolds number is from 100 to 1500 with an interval of 100. The existence of the one-cell, the two-cell and the twin-cell mode has confirmed numerically. The mode transitions have been observed when the Reynolds number is decreased.



(a) Experimental result ( $\Gamma = 0.51$ ,  $Re = 653$ )  
 (b) Numerical result ( $\Gamma = 0.6$ ,  $Re = 1400$ )

Figure 4. Twin-cell mode

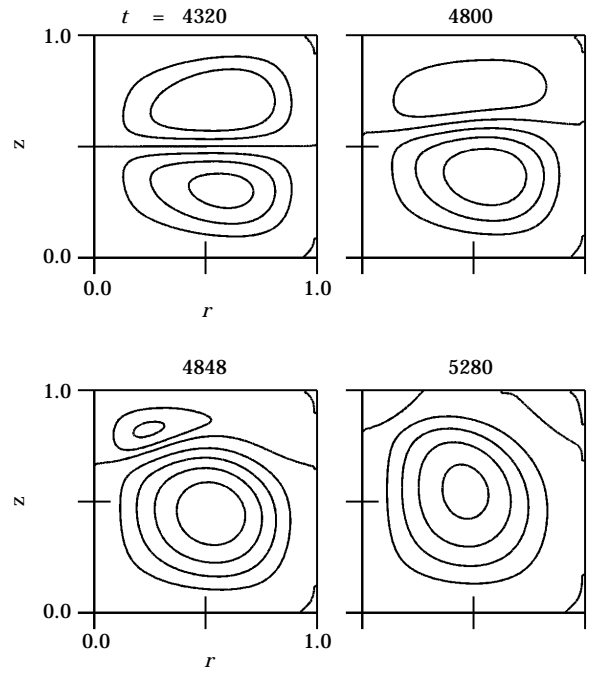


Figure 6. Development of flow field from normal two-cell mode to anomalous one-cell mode. The aspect ratio is 1.0 and the Reynolds number is reduced from 800 to 500. Deceleration starts at  $t = 2400$  and ends at  $t = 4800$ .

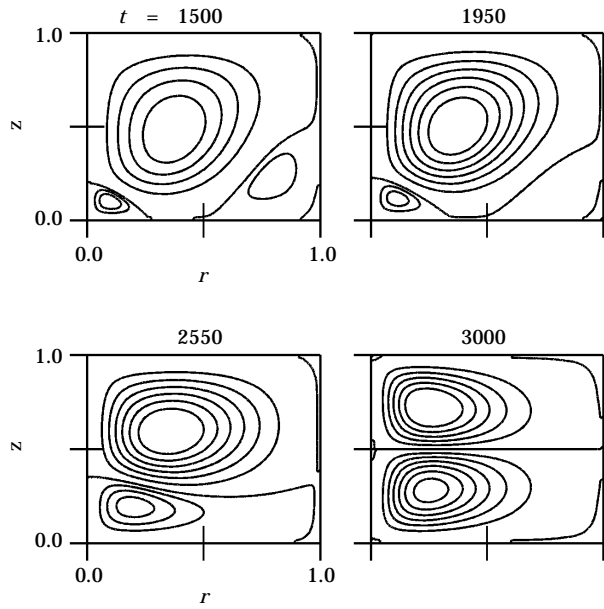


Figure 5. Development of flow field from anomalous one-cell mode to normal two-cell mode. The aspect ratio is 0.8 and the Reynolds number is reduced from 300 to 200. Deceleration starts at  $t = 900$  and ends at  $t = 1800$ .

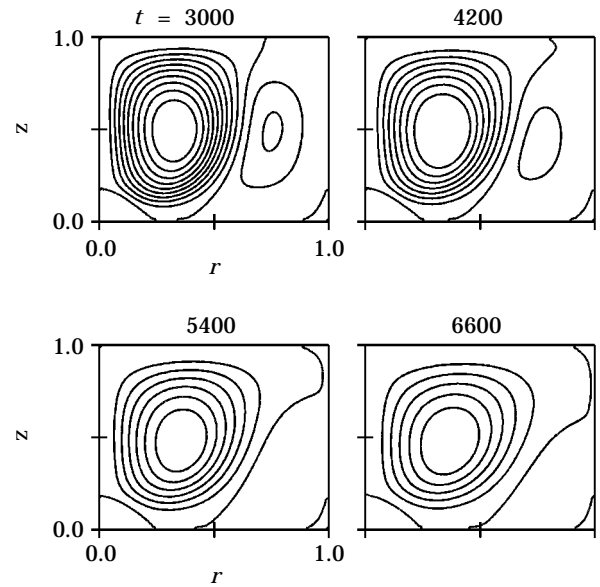


Figure 7. Development of flow field from twin-cell mode to anomalous one-cell mode. The aspect ratio is 0.8 and the Reynolds number is reduced from 1000 to 600. Deceleration starts at  $t = 3000$  and ends at  $t = 6000$ .